Surface Areas and Volumes

Assertion & Reason Type Questions

In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct option:

a. Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A)

b. Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A)

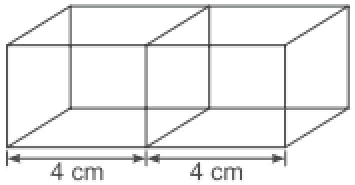
c. Assertion (A) is true but Reason (R) is false

d. Assertion (A) is false but Reason (R) is true

Q 1. Assertion (A): Suppose two equal cubes of edge 4 cm are joined together then the surface area of resulting cuboid is 160 cm^2.

Reason (R): We combined two equal cubes of edge a cm, then the length of the resulting cuboid will be 2a cm.

Answer : (a) **Assertion (A):** Given edge of a cube is 4 cm.



Here, length, 12 x 48 cm,

width, b4 cm

and height, h=4 cm

:- The surface area of resulting cuboid

= 2 (lb+bh + hl)

=2(8x4+4x4+4x8)

=2(32+16+32)=2x80=160 cm^2

So, Assertion (A) is correct.

Reason (R): It is true to say that length of the combined cubes is 2a cm.

Hence, both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

Q 2. Assertion (A): The radii of two cones are in the ratio 2:3 and their volumes in the ratio 1 : 3. Then the ratio of their heights is 3: 4.
Reason (R): Volume of a cone can be determined by

the formula, $V = \frac{1}{3}\pi r^2 h.$

Answer :

(a) **Assertion (A):** Let r_1 and r_2 be the radii of two cones and h, and h_2 be the heights of two cones. Then

$$\frac{r_1}{r_2} = \frac{2}{3} \qquad \text{...(1)}$$

Now, ratio of two volumes is $\frac{V_1}{V_2} = \frac{1}{3}$

$$\Rightarrow \quad \frac{\frac{1}{3}\pi r_1^2 h_1}{\frac{1}{3}\pi r_2^2 h_2} = \frac{1}{3} \quad \Rightarrow \quad \left(\frac{r_1}{r_2}\right)^2 \times \left(\frac{h_1}{h_2}\right) = \frac{1}{3}$$

$$\Rightarrow \quad \left(\frac{2}{3}\right)^2 \times \left(\frac{h_1}{h_2}\right) = \frac{1}{3} \qquad (\because \text{ from eq. (1)})$$

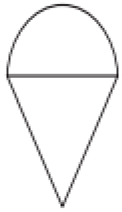
$$\Rightarrow \quad \frac{h_1}{h_2} = \frac{1}{3} \times \frac{9}{4} = \frac{3}{4}$$

So, Assertion (A) is true.
Reason (R): It is true that volume of any cone can be determined by the formula

$$V = \frac{1}{3}\pi r^2 h$$

Hence, both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

Q 3. Assertion (A): Total surface area of the toy is the sum of the curved surface area of the hemisphere and the curved surface area of the cone.
Reason (R): Toy is obtained by fixing the plane surfaces of the hemisphere and cone together.



Answer : (a) **Assertion (A):** We know that, the surface area of combined figure is the sum of all the curved surface of the individual figures. TSA of the toy = CSA of hemisphere + CSA of the cone.
So, Assertion (A) is true.
Reason (R): It is also a true statement because any combined figure is obtained by fixing the plane surfaces of individual figures. So, Reason (R) is true.
Hence, both Assertion (A) and Reason (R) is true and Reason (R) is the correct explanation of Assertion (A).

Q 4. Assertion (A): If the radius of a cone is halved and volume is not changed, then height remains same.
Reason (R): If the radius of a cone is halved and volume is not changed then height must become four times of the original height.

Answer : (d) **Assertion (A):** If the radius of a cone is halved and volume is not changed, then height is not remain same.
So, Assertion (A) is false.
Reason (R): Let V_1 and V_2 be the volume of two cones. Then

$$\frac{V_1}{V_2} = \frac{\frac{1}{3}\pi r_1^2 h_1}{\frac{1}{3}\pi r_2^2 h_2}$$

$$\frac{V_1}{V_2} = \frac{r^2 h_1}{\left(\frac{r}{2}\right)^2 h_2}$$

But V_1 and V_2 are equal.

$$\therefore \quad r^2 h_1 = \left(\frac{r}{2}\right)^2 h_2$$

$$\Rightarrow \quad h_1 = \frac{1}{4} h_2$$

$$\Rightarrow \quad 4h_1 = h_2$$

So, Reason (R) is true.
Hence, Assertion (A) is false but Reason (R) is true.

Q 5. Assertion (A): If the volumes of two spheres are in the ratio 216: 125. Then their surface areas are in the ratio 6: 5.

Reason (R): Volume of the sphere $= \frac{4}{3}\pi r^3$ **and its surface area** $= 4\pi r^2$.

Answer : (b) **Assertion (A):** Let V_1 and V_2 be the volumes of two spheres. Also, let r_1 and r_2 be the radius of two spheres respectively. Then

$$\frac{V_1}{V_2} = \frac{216}{125}$$

$$\Rightarrow \quad \frac{\frac{4}{3}\pi r_1^3}{\frac{4}{3}\pi r_2^3} = \left(\frac{6}{5}\right)^3$$

$$\Rightarrow \quad \left(\frac{r_1}{r_2}\right)^3 = \left(\frac{6}{5}\right)^3$$

$$\Rightarrow \quad \frac{r_1}{r_2} = \frac{6}{5}$$

$\therefore$ The ratio of surface areas of two spheres is

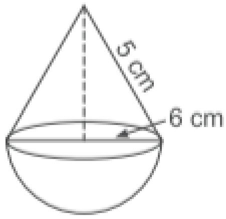
$$\frac{S_1}{S_2} = \frac{4\pi r_1^2}{4\pi r_2^2} = \left(\frac{r_1}{r_2}\right)^2 = \left(\frac{6}{5}\right)^2 = \frac{36}{25}$$

Reason (R): It is also a true statement.

Hence, both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

Q 6. Assertion (A): The surface area of the combined figure is 320.28 cm^2. [Use л = 3.14]

Reason (R): The surface area of combined figure is the difference of curved surface areas of individual figures.



Answer : (c) **Assertion (A):** Given, slant height of cone, l = 5 cm,

radius of cone = radius of hemisphere r = 6 cm.

:- Surface area of the combined figure

= CSA of cone + CSA of hemisphere

= $\pi rl + 2\pi r^2$

$= 3.14 \times 6 \times 5 + 2 \times 3.14 \times (6)^2$

$= 94.2 + 226.08$

$= 320.28 \text{ cm}^2$

So, Assertion (A) is true.

Reason (R): It is a false statement. The right statement is that the surface area of the combined figure is the sum of curved surface areas of individual figures.

Hence, Assertion (A) is true but Reason (R) is false.

Q.7**. Assertion (A):** Total surface area of the cylinder having radius of the base 14 cm and height 30 cm is 3872 cm^2.

Reason (R): If r be the radius and h be the height of the cylinder, then total surface area = $(2\pi rh + 2\pi r^2)$.

Answer : (a) Total surface area $= 2\pi rh + 2\pi r^2$

$$= 2\pi r(h + r)$$

$$= 2 \times \frac{22}{7} \times 14(30 + 14) = 88(44)$$

$$= 3872 \text{ cm}^2$$

Q.8**. Assertion (A):** The slant height of the frustum of a cone is 5 cm and the difference between the radii of its two circular ends is 4 cm. Than the height of the frustum is 3 cm.

Reason (R): Slant height of the frustum of the cone is given by

$l = \sqrt{(R - r)^2 + h^2}$.

Answer : (a) We have, $l = 5 \text{ cm},\ R - r = 4 \text{ cm}$

$$5 = \sqrt{(4)^2 + h^2}$$

$$16 + h^2 = 25$$

$$h^2 = 25 - 16 = 9$$

$$h = 3 \text{ cm}$$

Q.9. Assertion (A): If the height of a cone is 24 cm and diameter of the base is 14 cm, then the slant height of the cone is 15 cm.

Reason (R): If r be the radius and h the slant height of the cone, then slant height $= \sqrt{h^2 + r^2}$.

Answer : (d) Slant height $= \sqrt{(14/2)^2 + (24)^2}$

$$= \sqrt{49 + 576}$$

$$= \sqrt{625} = 25$$

Q.10. Assertion (A): Two identical solid cube of side 5 cm are joined end to end. Then total surface area of the resulting cuboid is 300 cm^2 .

Reason (R): Total surface area of a cuboid is 2 (lb + bh + lh)

Answer : (d) When cubes are joined end to end, it will form a cuboid.

l = 2 x 5 = 10 cm, b= 5 cm

and h = 5 cm

Total surface area = 2(lb + bh + lh)

= 2(10 x 5+5 X 5+10 x 5)

= 2 X 125 = 250 cm^2

Q.11. Assertion (A): If the radius of a cone is halved and volume is not changed, then height remains same.

Reason (R): If the radius of a cone is halved and volume is not changed then height must become four times of the original height.

Answer : (d)

$$\frac{V_1}{V_2} = \frac{(1/3)\pi r^2 h_1}{(1/3)\pi (r/2)^2 h_2} = \frac{4h_1}{h_2}$$

As $V_1 = V_2$

$$h_2 = 4h_1$$

Q.12**. Assertion (A):** The radii of two cones are in the ratio 2:3 and their volumes in the ratio 1:3. Then the ratio of their heights is 3:2.

Reason (R): Volume of the cone $= \frac{1}{3}\pi r^2.h$

Answer : (d)

We have, ratio of volume $= \dfrac{\frac{1}{3}\pi \times (2x)^2 \times h_1}{\frac{1}{3}\pi \times (3x)^2 \times h_2}$

$$\frac{1}{3} = \frac{4}{9} \times \frac{h_1}{h_2}$$

$$\frac{h_1}{h_2} = \frac{3}{4}$$

$$h_1 : h_2 = 3:4$$

Q.13. Assertion (A): If a ball is in the shape of a sphere has a surface area of 221.76 cm^2 , then its diameter is 8.4 cm.

Reason (R): If the radius of the sphere be r , then surface area,

$S = 4\pi r^2$, i.e. $r = \frac{1}{2}\sqrt{\frac{S}{\pi}}$.

Answer : (a)

Q.14**. Assertion (A):** The number of coins 1.75 cm in diameter and 2 mm thick is formed from a melted cuboid 10cm × 5.5cm × 3.5cm is 400.

Reason (R): Volume of a cylinder $=\pi r^2 h$ cubic units and area of cuboid = (l × b × h) cubic units.

Answer : (a)

$$\text{Number of coins} = \frac{\text{volume of cuboid}}{\text{volume of one coin}}$$

$$= \frac{10 \times 5.5 \times 3.5}{\pi \times \frac{1.75}{2} \times \frac{1.75}{2} \times 0.2}$$

$$\frac{10 \times 5.5 \times 3.5}{\frac{22}{7} \times \frac{1.75}{2} \times \frac{1.75}{2} \times 0.2} = 400$$

Q.15**. Assertion (A):** No. of spherical balls that can be made out of a solid cube of lead whose edge is 44 cm, each ball being 4 cm. in diameter, is 2541

Reason (R): $\text{Number of balls} = \frac{\text{Volume of one ball}}{\text{volume of lead}}$

Answer : (c)

Q.16**. Assertion (A):** If the volumes of two spheres are in the ratio 27:8. Then their surface areas are in the ratio 3:2.

Reason (R): Volume of the sphere $= \frac{4}{3}\pi r^3$ and its surface area = $4\pi r^2$

Answer : (d)

$$\text{We have,} \quad \frac{\frac{4}{3}\pi R^3}{\frac{4}{3}\pi r^3} = \frac{27}{8}$$

$$\frac{R^3}{r^3} = \frac{27}{8}$$

$$\frac{R}{r} = \frac{3}{2}$$

$$\text{Ratio of surface area} = \frac{4\pi R^2}{4\pi r^2} = \frac{R^2}{r^2} = \left(\frac{3}{2}\right)^2 = \frac{9}{4}$$